Efficient Solvers for Field-Scale Simulation of Flow and Transport in Porous Media

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Part I

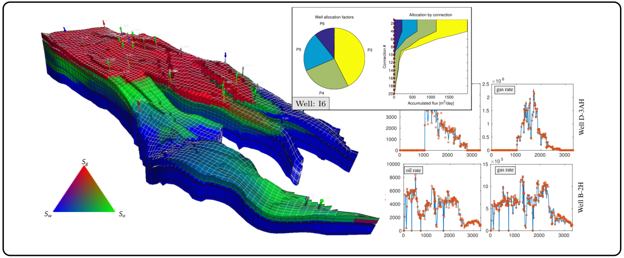
Introduction

Introduction

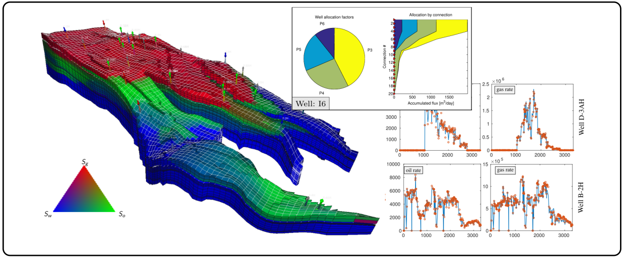
Flow in porous media

The study of fluid flow (water, oil, gas) through pores in a solid (porous rock formation)

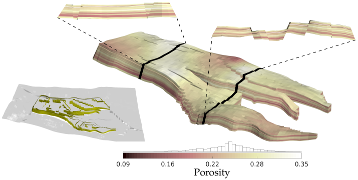
- Hydrocarbon recovery from petroleum reservoirs
  - Fossil fuels and petrochemical products (e.g., lubricants, fertilizers, plastics)
- Geothermal energy exploitation
  - Harness the thermal energy from underground aquifers
- CO_2 storage to mitigate greenhouse effects
  - Capture CO_2 from industrial processes and store it underground



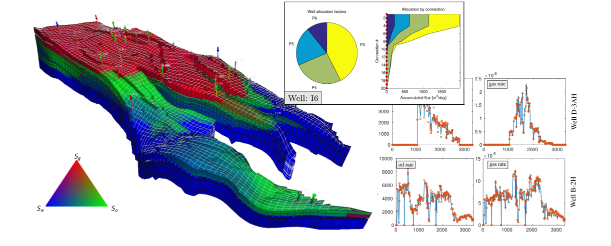
Well allocation factors
P6
P5
P3
P4
Allocation by connection
Connection #
Accumulated flux [m³/day]
Well: I6
gas rate
Well D-3AH
oil rate
gas rate
Well B-2H



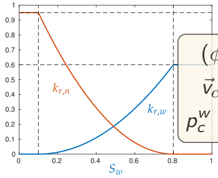
Geological model



Introduction – Reservoir Simulation



Flow model

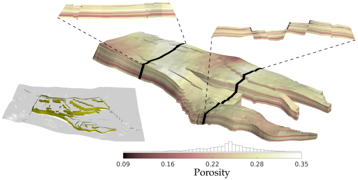


$$(\phi\rho_\alpha S_\alpha)_t + \nabla\cdot(\rho_\alpha\vec{v}_\alpha) = q_\alpha$$

$$\vec{v}_\alpha = -\lambda_\alpha \mathbf{K}(\nabla p_\alpha - \rho_\alpha g \nabla z)$$

$$p_c^w = p_n - p_w, \qquad S_w + S_n = 1$$

Geological model



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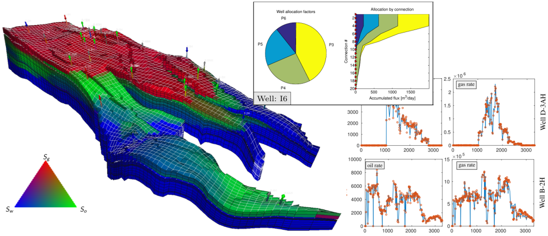
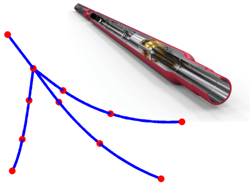
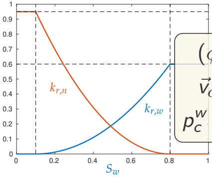


Well model



Flow model



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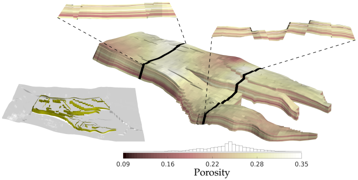
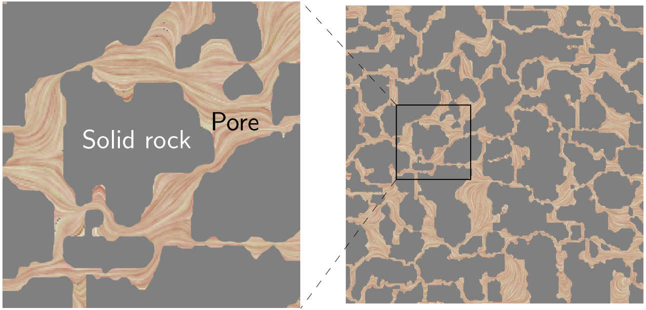
Geological model

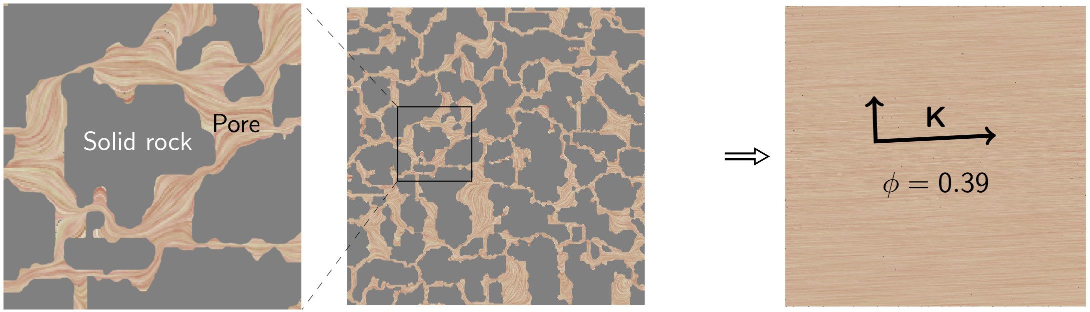


Flow in Porous Media – The Geological Model



- Flow in pore networks is complex, and requires extensive computer resources to simulate

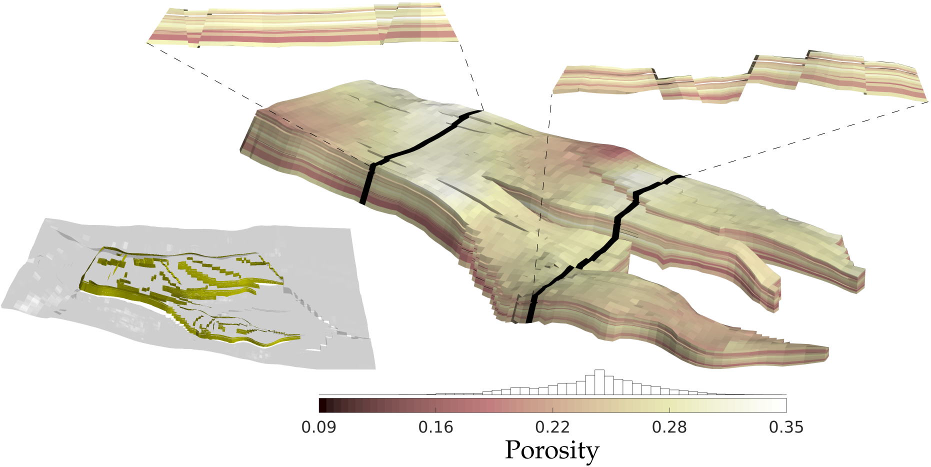
Flow in Porous Media – The Geological Model



- Flow in pore networks is complex, and requires extensive computer resources to simulate
- ... but we don't need this level of detail!
  - Instead: approximate porous rock by *representative elementary volume* (REV)

K: permeability – rock's ability to transmit a fluid | ϕ: porosity – fraction of rock that is pore space

Flow in Porous Media – The Geological Model



Norne oil and gas field in the the Norwegian Sea, operated by Equinor

Flow in Porous Media – The Flow Model

- Conservation of mass of fluid phase α on semi-discrete, implicit, residual form

$$\mathcal{R}_\alpha^{n+1} = \frac{1}{\Delta t^n}(\mathcal{M}_\alpha^{n+1} - \mathcal{M}_\alpha^{n}) + \nabla \cdot \vec{\mathcal{F}}_\alpha^{n+1} - \mathcal{Q}_\alpha^{n+1} = 0, \quad \alpha = a, \ell, v$$

Accumulation | Fluxes | Sources/sinks

- For immiscible multiphase flow, we have

$$\mathcal{M}_\alpha = \phi \rho_\alpha S_\alpha, \quad \vec{\mathcal{F}}_\alpha = \rho_\alpha \vec{v}_\alpha, \quad \mathcal{Q}_\alpha = \rho_\alpha q_\alpha$$

- Darcy velocity $\vec{v}_\alpha$ given by Darcy's law

$$\vec{v}_\alpha = -\lambda_\alpha \mathbf{K}(\nabla p_\alpha - \rho_\alpha g \nabla z), \quad \lambda_\alpha = \frac{k_{r,\alpha}}{\mu_\alpha}$$

- Closure relations

$$S_a + S_\ell + S_v = 1, \qquad p_c^\alpha = p_\ell - p_\alpha \quad \text{for } \alpha = a, v$$

Flow in Porous Media – The Flow Model

Sequential splitting – flow and transport

- Physical quantities in $\mathcal{R}_\alpha = 0$ exhibit very different mathematical character

$$\underbrace{\text{flow variables, e.g. } p, \vec{v}}_{\text{elliptic}} \qquad \underbrace{\text{transport variables, e.g. } S_\alpha}_{\text{hyperbolic}}$$

Flow in Porous Media – The Flow Model

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- Flow equation: weighted sum of conservation equations

$$\mathcal{R}_F^{n+1} = \sum_{\alpha=a,\ell,v} \omega_\alpha \mathcal{R}_\alpha^{n+1} = 0, \quad \text{where} \quad \sum_{\alpha=a,\ell,v} \partial_u(\omega_\alpha \mathcal{M}_\alpha^{n+1}) = 0 \quad \text{for } u \neq p$$

Sequential splitting – flow and transport

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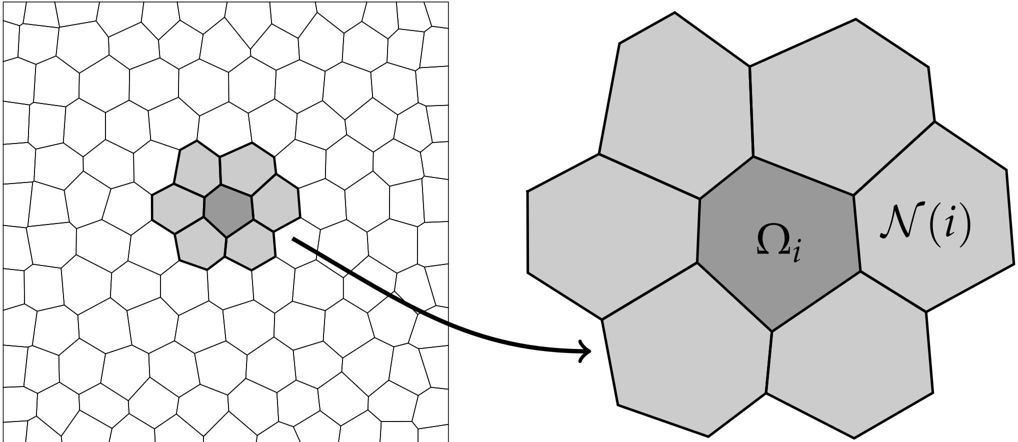
- Transport equations: $\mathcal{R}_\alpha = 0$ with $\vec{v}_\alpha$ redefined with total velocity $\vec{v} = \vec{v}_a + \vec{v}_\ell + \vec{v}_v$

$$\vec{v}_\alpha = f_\alpha \Big(\vec{v} + \mathbf{K} \sum_{\beta=a,\ell,v} \lambda_\beta [\vec{G}_\alpha - \vec{G}_\beta] \Big), \quad f_\alpha = \frac{\lambda_\alpha}{\lambda_a + \lambda_\ell + \lambda_v} \quad \text{and} \quad \vec{G}_\alpha = \rho_\alpha g \nabla z - \nabla p_c^\alpha$$

Discretization



Discretization

Spatial discretization

- Integrate residual equations over each cell in space → finite-volume discretization

$$\mathcal{R}_\alpha^{n+1} = \frac{1}{\Delta t^n}(\mathcal{M}_\alpha^{n+1} - \mathcal{M}_\alpha^{n}) + \nabla \cdot \mathcal{F}_\alpha^{n+1} - \mathcal{Q}_\alpha^{n+1} = 0$$

Discretization

Spatial discretization

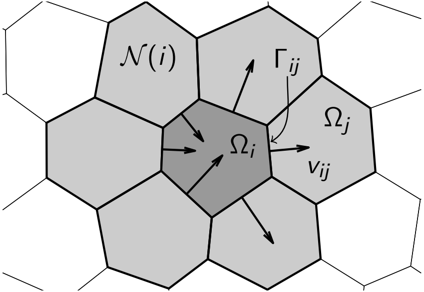
- Integrate residual equations over each cell in space → finite-volume discretization

$$\int_{\Omega_i} \mathcal{R}_\alpha^{n+1} \mathrm{d}V = \frac{1}{\Delta t^n} \int_{\Omega_i} (\mathcal{M}_\alpha^{n+1} - \mathcal{M}_\alpha^n) \mathrm{d}V + \int_{\Omega_i} \nabla \cdot \vec{\mathcal{F}}_\alpha^{n+1} \mathrm{d}V - \int_{\Omega_i} \mathcal{Q}_\alpha^{n+1} \mathrm{d}V = 0$$

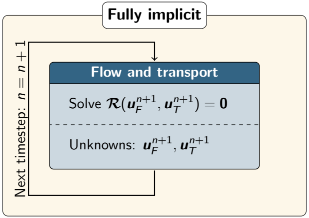
$$\int_{\Omega_i} \mathcal{M}_\alpha \mathrm{d}V \approx |\Omega_i| \mathcal{M}_{\alpha,i} \qquad \text{Mass terms}$$

$$\int_{\Omega_i} \nabla \cdot \vec{\mathcal{F}}_\alpha \mathrm{d}V \approx \sum_{j \in \mathcal{N}(i)} \mathcal{F}_{\alpha,ij} \qquad \text{Flux terms}$$

$$\int_{\Omega_i} \mathcal{Q}_{\alpha,i} \mathrm{d}V \approx |\Omega_i| \mathcal{Q}_{\alpha,i} \qquad \text{Source terms}$$



Solution Strategies



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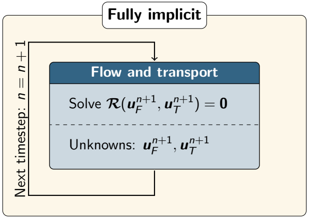
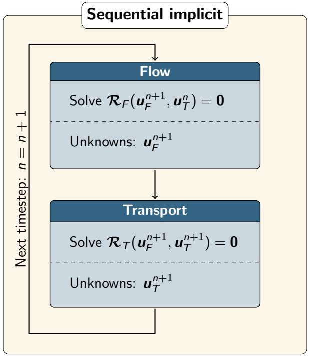
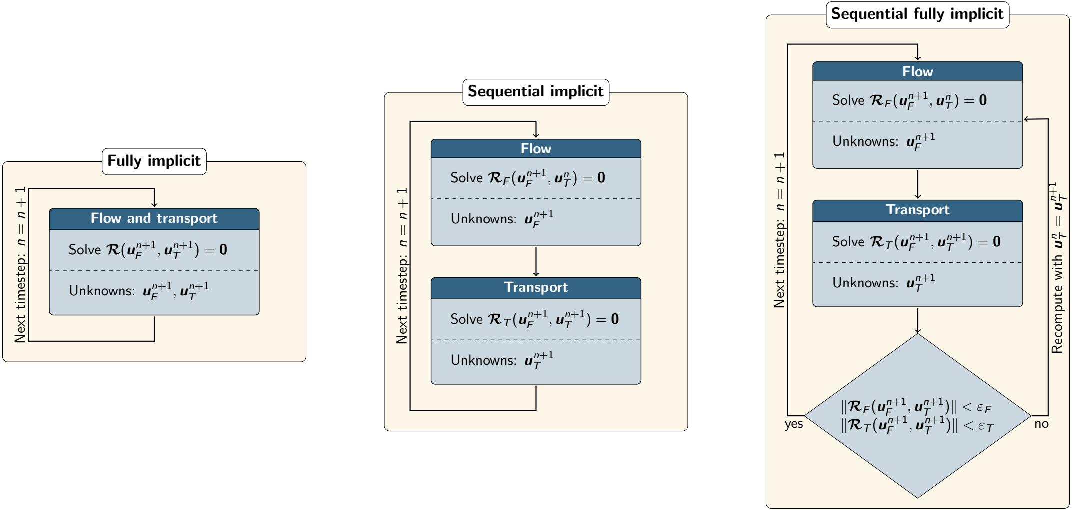




Solution Strategies



Solution Strategies – Newton's Method

Each strategy involves solving system of *nonlinear* residual equations $\boldsymbol{\mathcal{R}}(\boldsymbol{u}) = \mathbf{0}$

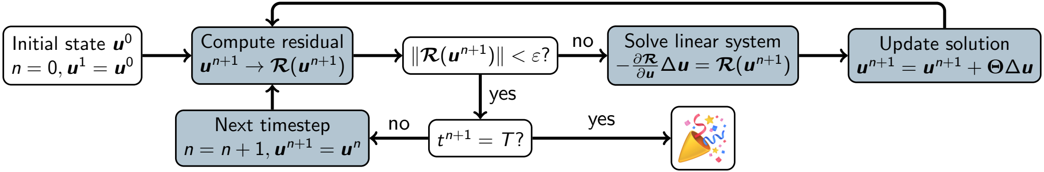
- Assume $\boldsymbol{\mathcal{R}}(\boldsymbol{u} + \Delta\boldsymbol{u}) = \mathbf{0}$, and linearize around $\boldsymbol{u}$

$$\mathbf{0} = \boldsymbol{\mathcal{R}}(\boldsymbol{u} + \Delta\boldsymbol{u}) = \boldsymbol{\mathcal{R}}(\boldsymbol{u}) + \frac{\partial \boldsymbol{\mathcal{R}}}{\partial \boldsymbol{u}}\Delta\boldsymbol{u} + \mathcal{O}(\|\Delta\boldsymbol{u}\|^2)$$

- Neglect higher-order terms $\rightarrow$ Newton's method

$$\boldsymbol{u}^{k+1} = \boldsymbol{u}^{k} + \Delta\boldsymbol{u}, \quad \text{where} \quad -\frac{\partial \boldsymbol{\mathcal{R}}}{\partial \boldsymbol{u}}\Delta\boldsymbol{u} = \boldsymbol{\mathcal{R}}(\boldsymbol{u}^{k})$$

Solution Strategies – Newton’s Method



Part II

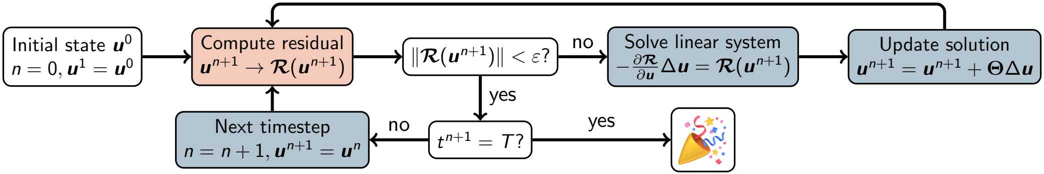
Scientific Papers

Paper I – III: Unstructured Gridding and Consistent Discretizations

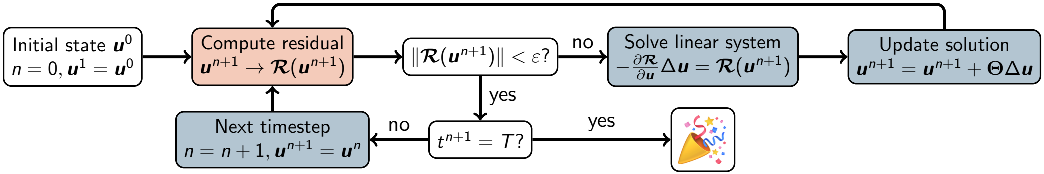
I: **Unstructured Gridding and Consistent Discretizations for Reservoirs With Faults and Complex Wells**
Øystein S. Klemetsdal, Runar Lie Berge, Knut-Andreas Lie, Halvor Møll Nilsen, Olav Møyner
In proceedings of the 2017 SPE Reservoir Simulation Conference, Montgomery, Texas, USA
DOI: 10.2118/182666-MS

II: **Unstructured Voronoi Grids Conforming to Lower-dimensional Objects**
Runar Lie Berge, Øystein S. Klemetsdal, Knut-Andreas Lie
Computational Geosciences, volume 23, issue 1, pp. 169–188, 2019
DOI: 10.1007/s10596-018-9790-0

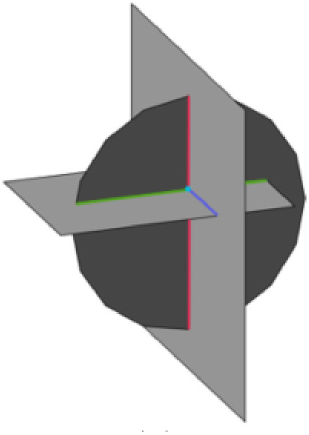
III: **A Comparison of Consistent Discretizations for Elliptic Poisson-Type Problems on Unstructured Polyhedral Grids**
Øystein S. Klemetsdal, Olav Møyner, Xavier Raynaud, Knut-Andreas Lie
Manuscript in preparation, 2019

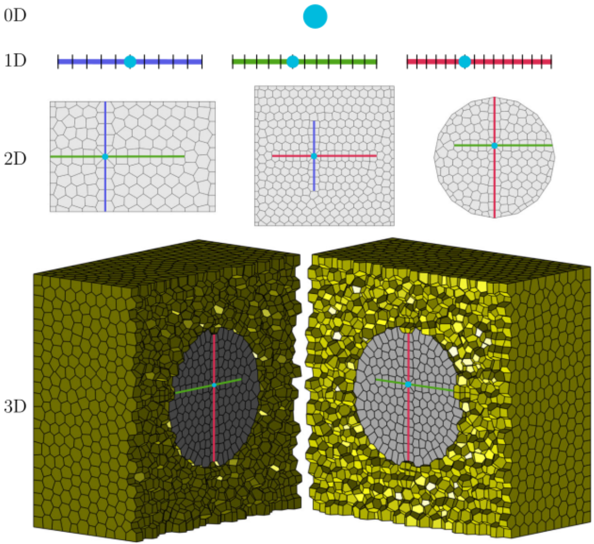


- The computational grid has a direct impact on the quality of the numerical solution
  - Conform to intersecting faults, fractures, well trajectories
    [Branets et al., 2009, Manzoor et al., 2018, Toor et al., 2015] ...
- ... but what is the best computational grid will depend on the specific discretization
  - Linear/nonlinear two-point, multipoint, mimetic, virtual elements, etc.
    [Le Potier, 2009, Aavatsmark et al., 1994, Brezzi et al., 2005] ...



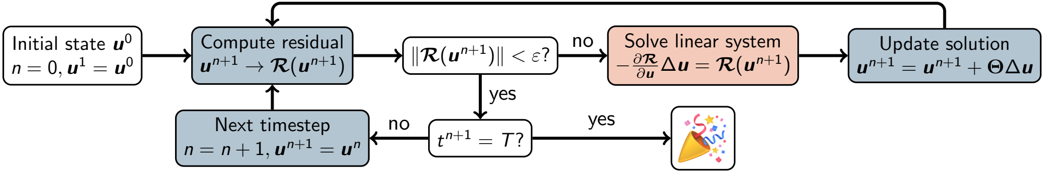
Paper I – III: Unstructured Gridding and Consistent Discretizations





Paper IV: Multiscale Simulation with Dynamically Adapted Basis Functions

IV: **Accelerating Multiscale Simulation of Complex Geomodels by Use of Dynamically Adapted Basis Functions**
Øystein S. Klemetsdal, Olav Møyner, Knut-Andreas Lie
Computational Geosciences, published ahead of print, 2019
DOI: 10.1007/s10596-019-9827-z



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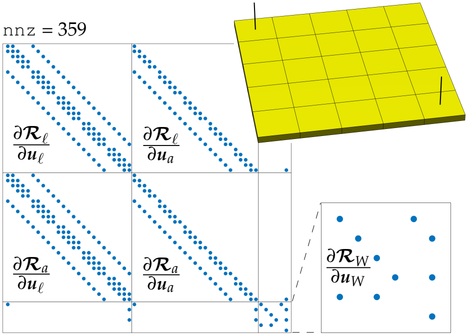
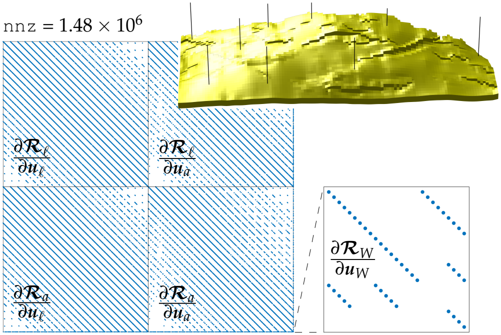


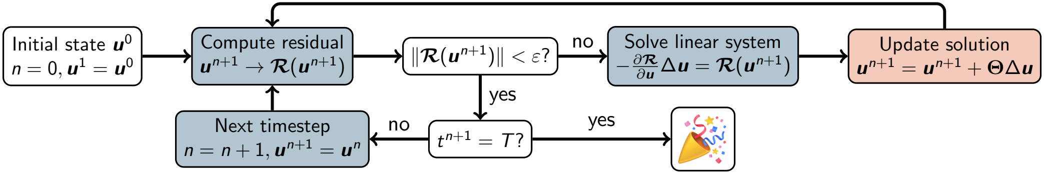


- Solving linearized systems typically accounts for a large portion of simulation time
  - Mixed elliptic/hyperbolic character $\rightarrow$ pressure is a strong variable
  - Large aspect ratios and variations in rock properties $\rightarrow$ ill-conditioned systems
- Efficient iterative linear solvers with efficient preconditioners are therefore crucial
  - Constrained pressure residual (CPR): physics-based preconditioner [Wallis et al., 1985]

Paper V–VI: Adaptive Interface-Localized Trust Region Solver

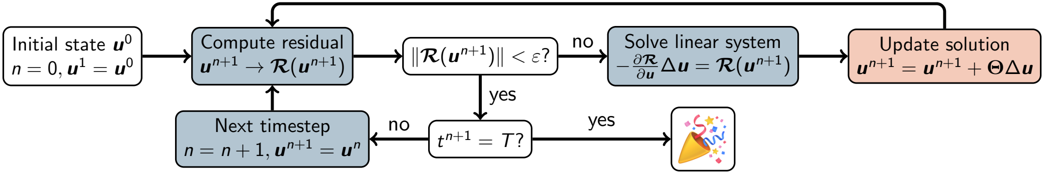
V: **Non-linear Newton Solver for a Polymer Two-phase System Using Interface-localized Trust Regions**
Øystein S. Klemetsdal, Olav Møyner, Knut-Andreas Lie
In proceedings of the 19th European Symposium on Improved Oil Recovery, 2017, Stavanger, Norway
DOI: 10.3997/2214-4609.201700356

VI: **Robust Nonlinear Newton Solver with Adaptive Interface-Localized Trust Regions**
Øystein S. Klemetsdal, Olav Møyner, Knut-Andreas Lie
SPE Journal, volume 24, issue 4, pp. 1576–1594, 2019
DOI: 10.2118/195682-PA



Paper V–VI: Adaptive Interface-Localized Trust Region Solver

- Transport problems are often very challenging for the nonlinear solver
  - Update $\Delta \boldsymbol{u}$ may send solution into different contraction regions
  - ... or cause changes in upstream direction
- Often caused by too long timestep
  - Whatever-works-approach: reduce timestep if solver has not converged after N iterations
  - Potentially large amount of wasted computational effort



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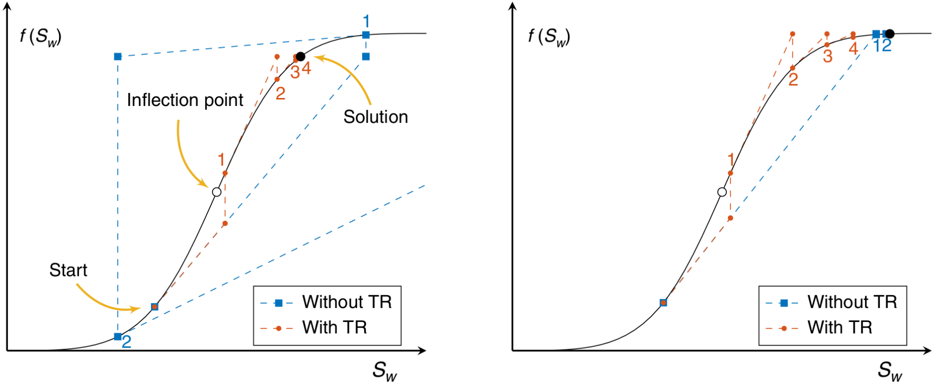
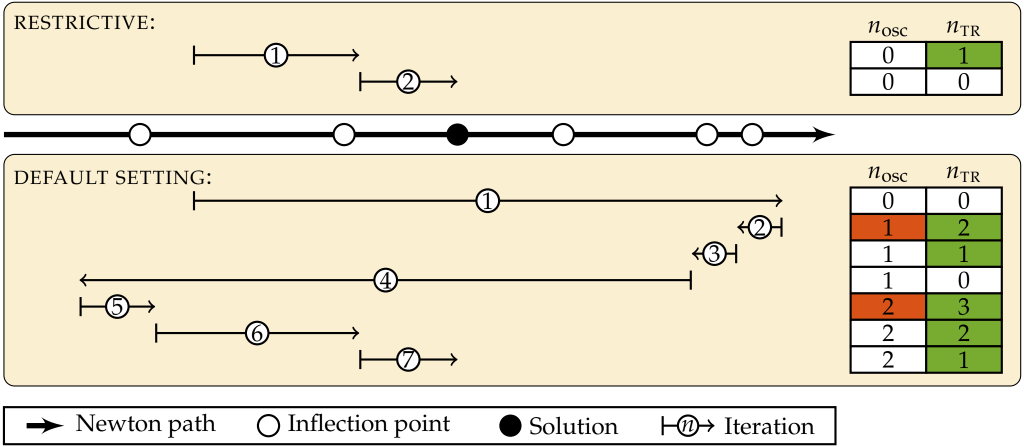


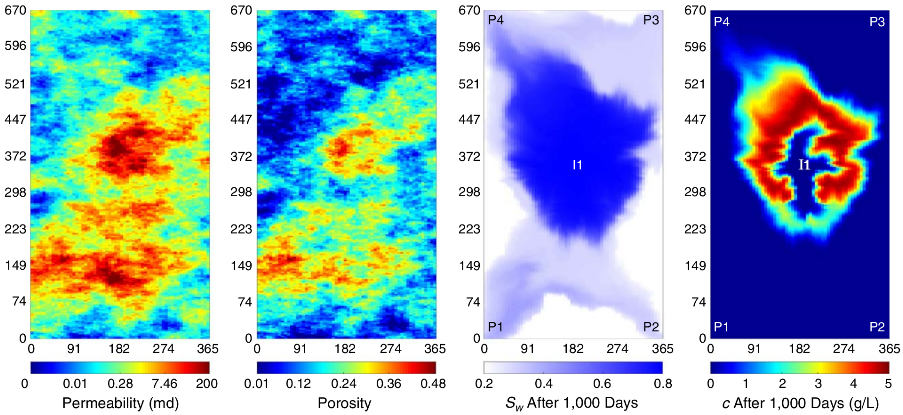
(a) Newton's method fails to converge (b) TR solver is overly restrictive

- Unconditional convergence by using trust regions [Jenny et al., 2009, Møyner, 2017]
  ... but computing trust regions is expensive, and damping may be overly restrictive

Paper V–VI: Adaptive Interface-Localized Trust Region Solver



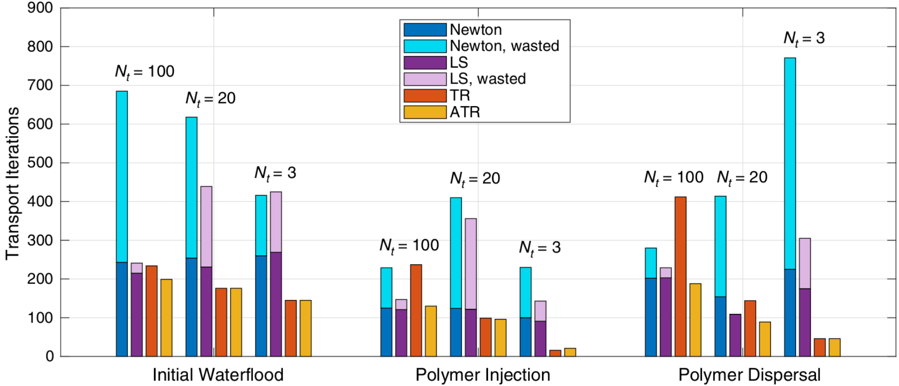
Example: Layer 10 of SEP10 Model 2



- Quadratic relative permeabilities, slightly compressible fluids/rock
- Simulate water + polymer slug + water over 2000 days using 100, 20, and 3 (!) timesteps
  - Water/polymer interplay + long timesteps challenging for nonlinear solver

Paper V–VI: Adaptive Interface-Localized Trust Region Solver

Example: Layer 10 of SEP10 Model 2



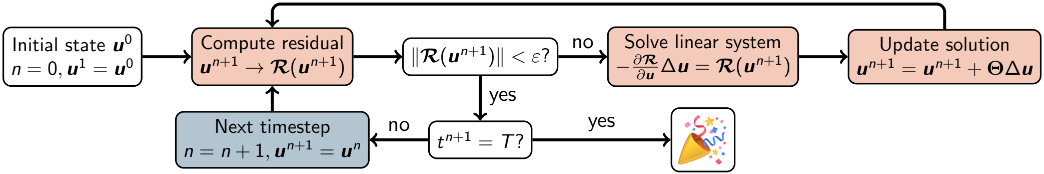
- Trust region: no wasted iterations even with only 3 timesteps
- Adaptive trust-region solver significantly better for modest timesteps

Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

VII: **Efficient Reordered Nonlinear Gauss-Seidel Solvers With Higher Order For Black-Oil Models**
Øystein S. Klemetsdal, Atgeirr Flø Rasmussen, Olav Møyner, Knut-Andreas Lie
Computational Geosciences, published ahead of print, 2019
DOI: 10.1007/s10596-019-09844-5

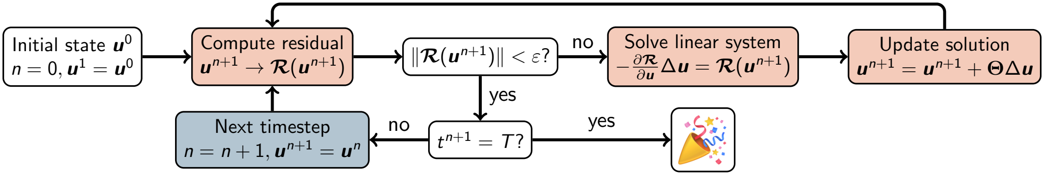
VIII: **Implicit High-resolution Compositional Simulation with Optimal Ordering of Unknowns and Adaptive Spatial Refinement**
Øystein S. Klemetsdal, Olav Møyner, Knut-Andreas Lie
In proceedings of the 2019 SPE Reservoir Simulation Conference, Galveston, Texas, USA
DOI: 10.2118/193934-MS

IX: **Dynamic Coarsening and Local Reordered Nonlinear Solvers for Simulating Transport in Porous Media**
Øystein S. Klemetsdal, Knut-Andreas Lie
Manuscript in preparation, 2019



Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

- Hyperbolic transport problems have *finite speed of propagation*
  - Updates $\Delta \boldsymbol{u}$ typically > 0 only near propagating fluid fronts and wells
  - Newton solver uses substantial efforts to compute zeros!
- Particularly true for real reservoir models: flow mainly restricted to drainage regions



Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

- Solve flow problem $\mathcal{R}_F = 0 \rightarrow$ pressure and intercell fluxes
- Split neighbors $\mathcal{N}(i)$ into upstream $\mathcal{U}(i)$ and downstream $\mathcal{D}(i)$

$$\frac{1}{\Delta t^n}(\mathcal{M}_i^{n+1} - \mathcal{M}_i^n) + \sum_{j\in\mathcal{N}(i)} \mathcal{F}_{ij}^{n+1} - \mathcal{Q}_i^{n+1} = 0$$

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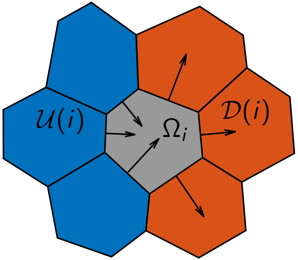

$$\frac{1}{\Delta t^n}(\mathcal{M}_i^{n+1} - \mathcal{M}_i^n) + \sum_{j \in \mathcal{U}(i)} \mathcal{F}_{ij}^{n+1} + \sum_{j \in \mathcal{D}(i)} \mathcal{F}_{ij}^{n+1} - \mathcal{Q}_i^{n+1} = 0$$

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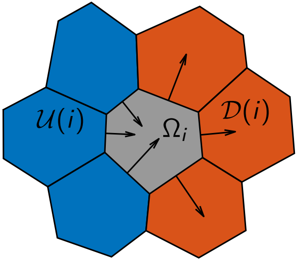
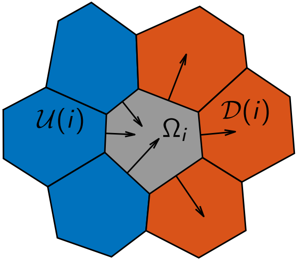
- Split neighbors $\mathcal{N}(i)$ into upstream $\mathcal{U}(i)$ and downstream $\mathcal{D}(i)$



$$\overbrace{\frac{1}{\Delta t^n}(\mathcal{M}_i^{n+1} - \mathcal{M}_i^n)}^{\text{depend on values in } \Omega_i} + \underbrace{\sum_{j \in \mathcal{U}(i)} \mathcal{F}_{ij}^{n+1}}_{\text{depend on values in } \mathcal{U}(i)} + \overbrace{\sum_{j \in \mathcal{D}(i)} \mathcal{F}_{ij}^{n+1} - \mathcal{Q}_i^{n+1}}^{\text{depend on values in } \Omega_i} = 0$$

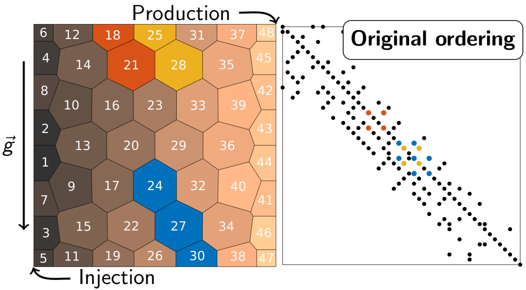
Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

- Solve flow problem $\mathcal{R}_F = 0 \rightarrow$ pressure and intercell fluxes
- Split neighbors $\mathcal{N}(i)$ into upstream $\mathcal{U}(i)$ and downstream $\mathcal{D}(i)$
- Only viscous forces: flux graph is acyclic (DAG)
  - Solve transport problems cell-by-cell in topological order [Natvig and Lie, 2008, Lie et al., 2014]

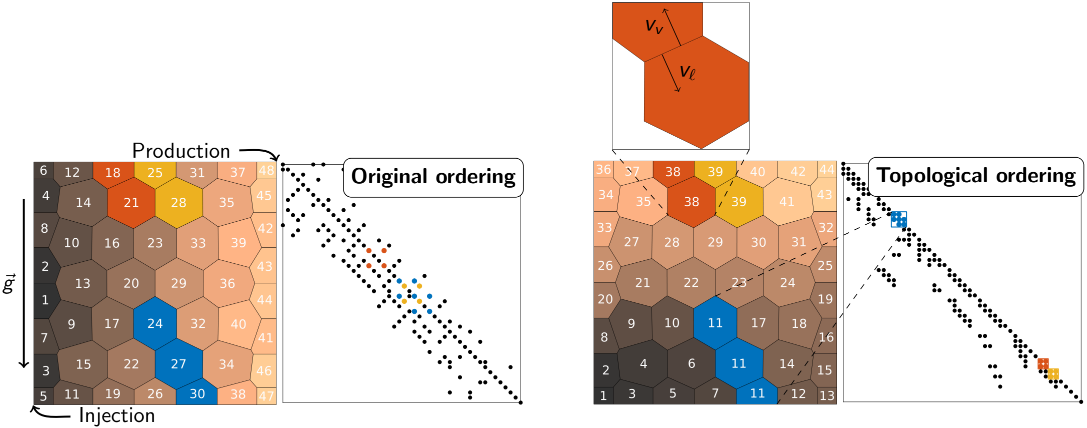


$$\overbrace{\frac{1}{\Delta t^n}(\mathcal{M}_i^{n+1} - \mathcal{M}_i^n)}^{\text{depend on values in } \Omega_i} + \underbrace{\sum_{j\in\mathcal{U}(i)} \mathcal{F}_{ij}^{n+1}}_{\text{depend on values in } \mathcal{U}(i)} + \overbrace{\sum_{j\in\mathcal{D}(i)} \mathcal{F}_{ij}^{n+1} - \mathcal{Q}_i^{n+1}}^{\text{depend on values in } \Omega_i} = 0$$

Paper VII–IX: Localized Reordered Nonlinear Transport Solvers



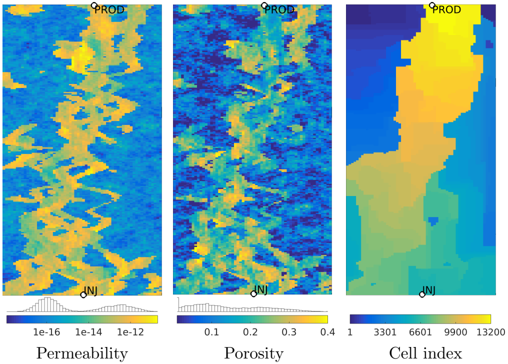
Paper VII–IX: Localized Reordered Nonlinear Transport Solvers



Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

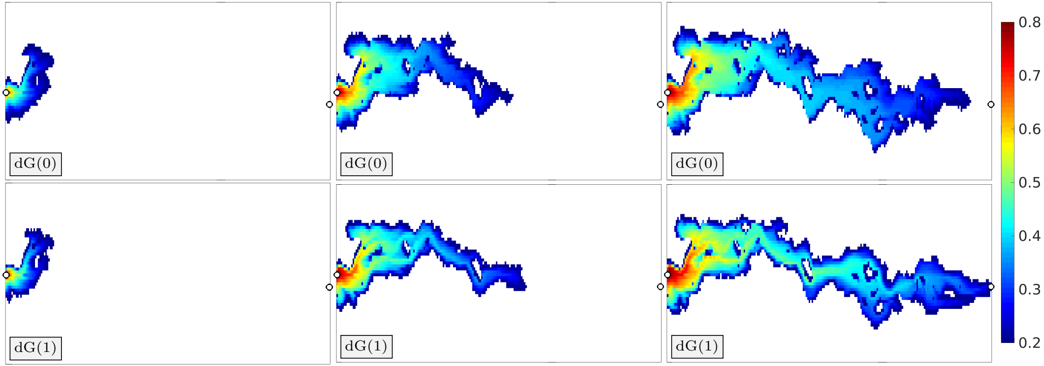
Example: Layer 50 of SPE 10 model 2

- Fluvial sandstone channels on mudstone
- Filled with oil, injection of 0.2 PV water
- Quadratic relative permeabilities
- Slightly compressible fluids/rock



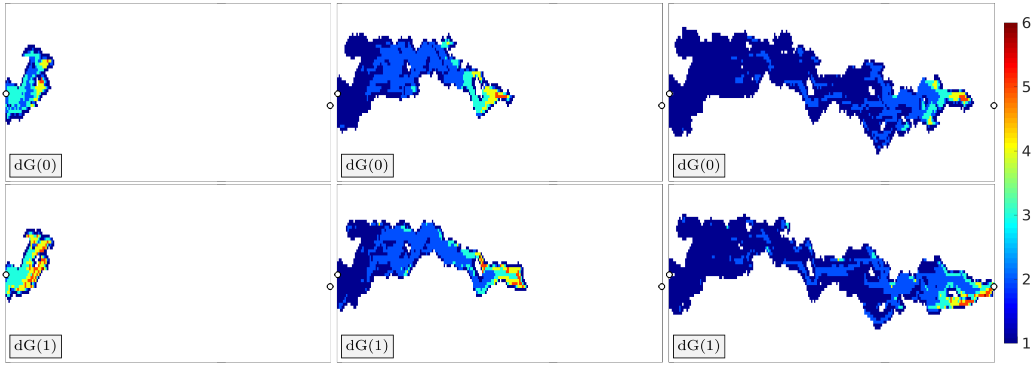
Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

Water saturation

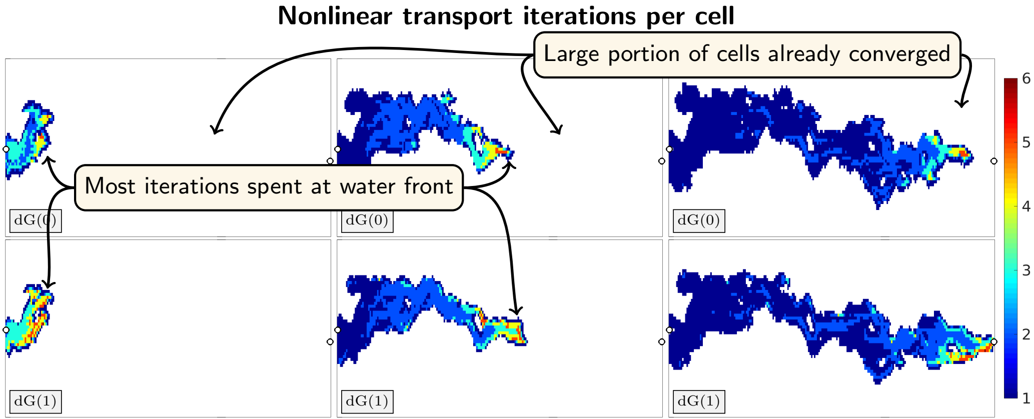


Paper VII–IX: Localized Reordered Nonlinear Transport Solvers

Nonlinear transport iterations per cell



Paper VII–IX: Localized Reordered Nonlinear Transport Solvers



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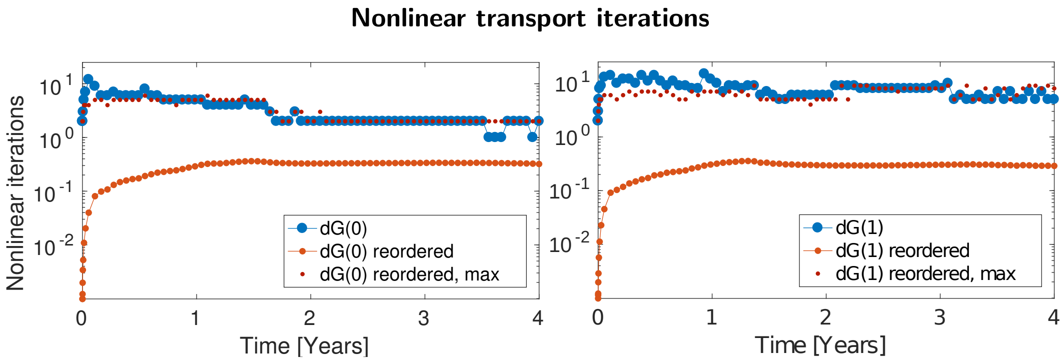
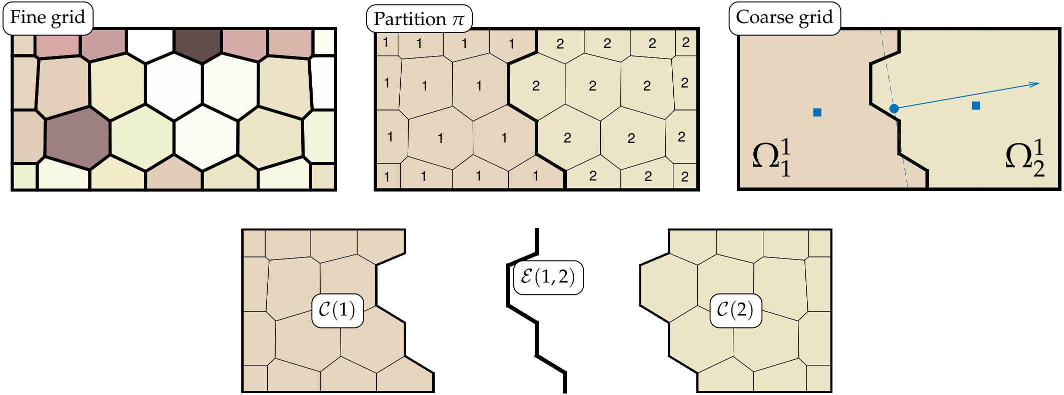
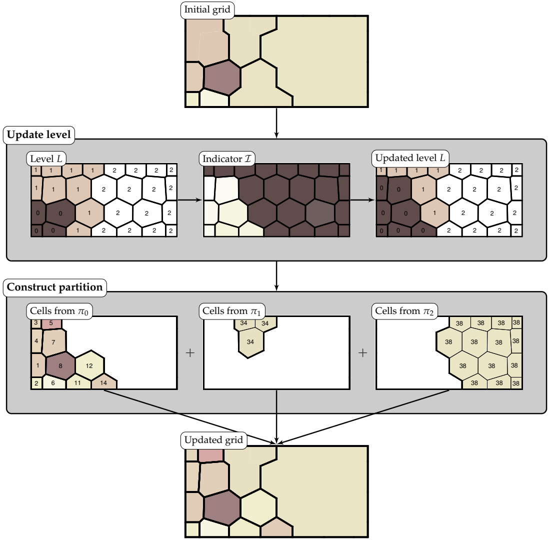


Paper XI: Dynamic Coarsening



- Dynamic grid refinement challenging for complex geomodels
- Construct coarse grids by partitioning (rectilinear, METIS, non-uniform coarsening, etc.)
  - Coarse grid block = aggregate of fine cells [Karypis and Kumar, 1998, Hauge et al., 2012]

Paper XI: Dynamic Coarsening



- Mapping should be mass conservative

$$|\Omega_i^a| \mathcal{M}_{\alpha,i}^a(u^a) = \sum_{j \in \mathcal{C}_a(i)} |\Omega_j| \mathcal{M}_{\alpha,j}(u) \qquad (1)$$

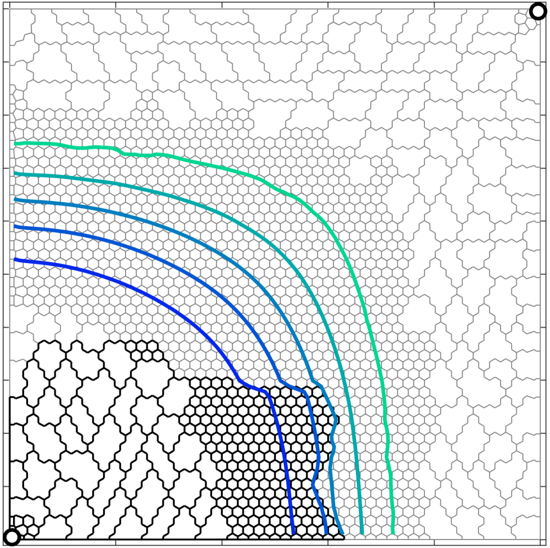
- Pressure and total intercell fluxes

$$\underbrace{p_{\alpha,i}^a = \frac{1}{\phi_i^a |\Omega_i^a|} \sum_{j \in \mathcal{C}_a(i)} \phi_j |\Omega_j| p_{\alpha,j},}_{\text{pore-volume-weighted}} \quad \underbrace{v_{ij}^a = \sum_{(m,n) \in \mathcal{E}_a(i,j)} v_{mn}}_{\text{sum fine-scale fluxes}}$$

- Immiscible: Saturations found by solving (1)
- After transport: map saturations to fine grid

$$S_{\alpha,j} = S_{\alpha,i}^a \quad \forall j \in \mathcal{C}(i),$$

Paper XI: Dynamic Coarsening



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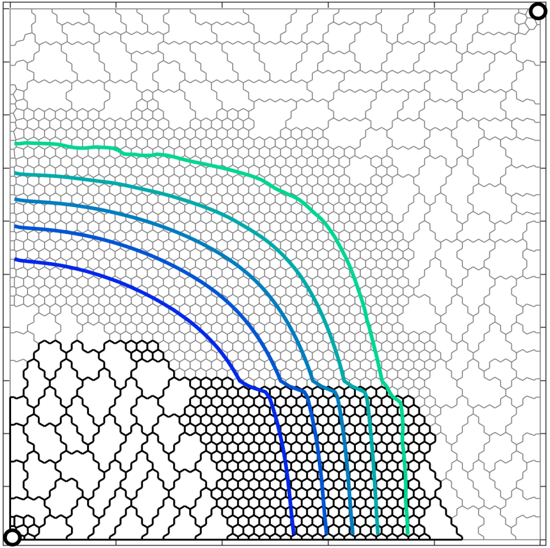

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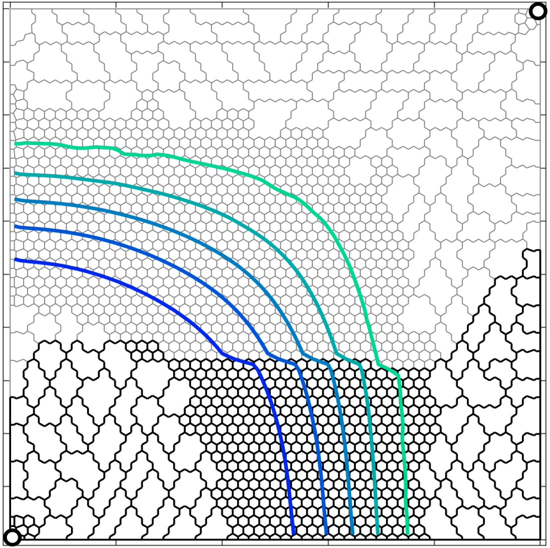

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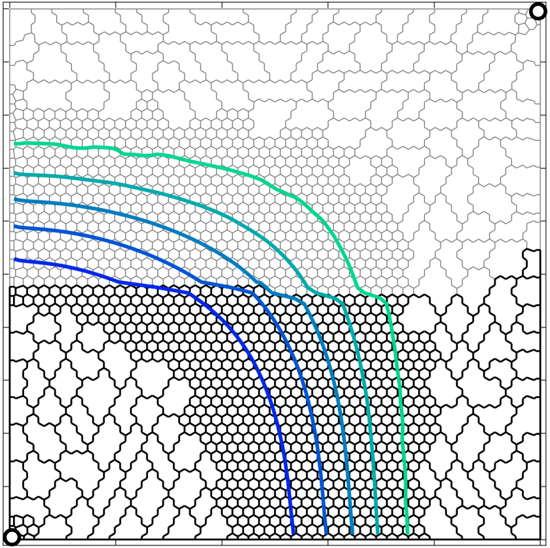

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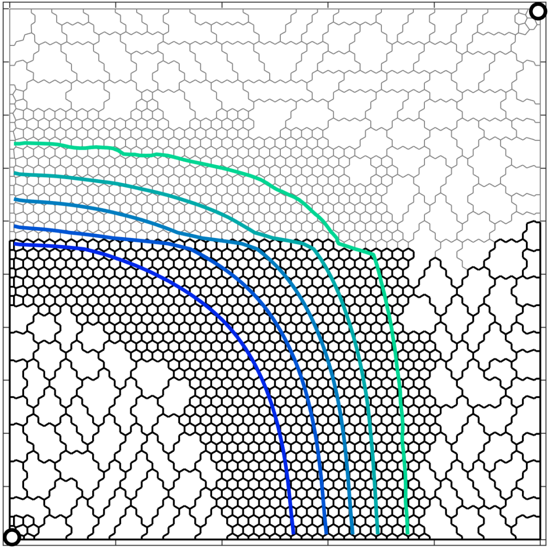

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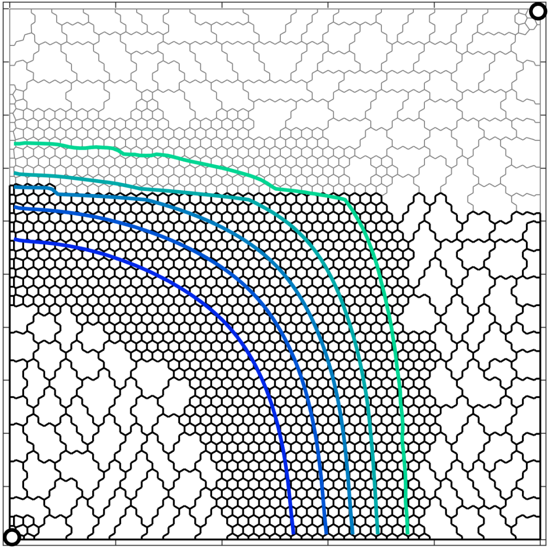

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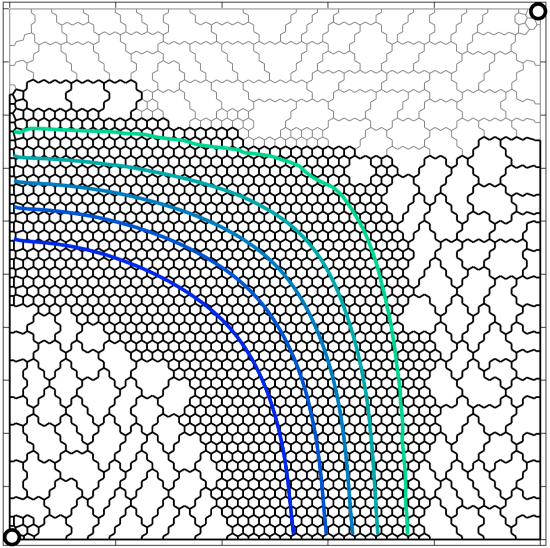

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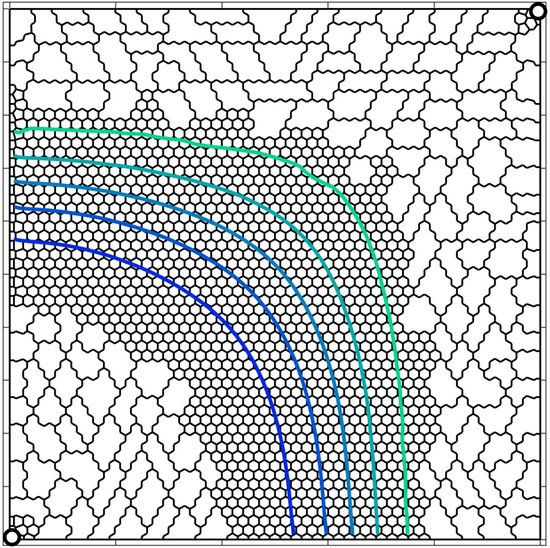

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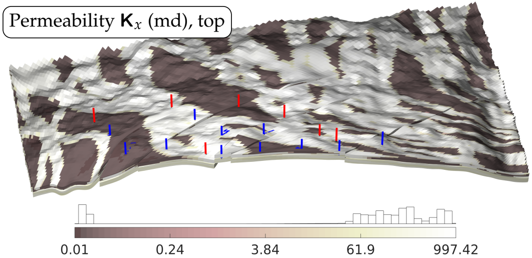

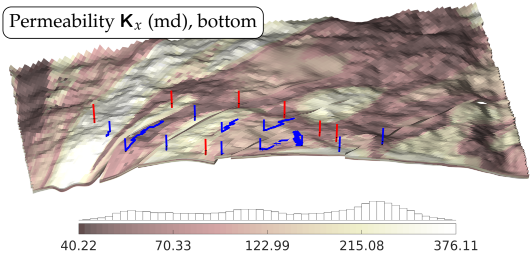

Example: Olympus field model [Fonseca et al., 2018]

- Cornerpoint grid format with 197 750 active cells, modelled from North Sea oil field
- Permeability/porosity: 1000 md/0.35 in sandstone channels – 1 md/0.03 in shale layers
- Compressible oil-water model: density 850/1020 kg/m^3, viscosity 2.59/0.395 cP

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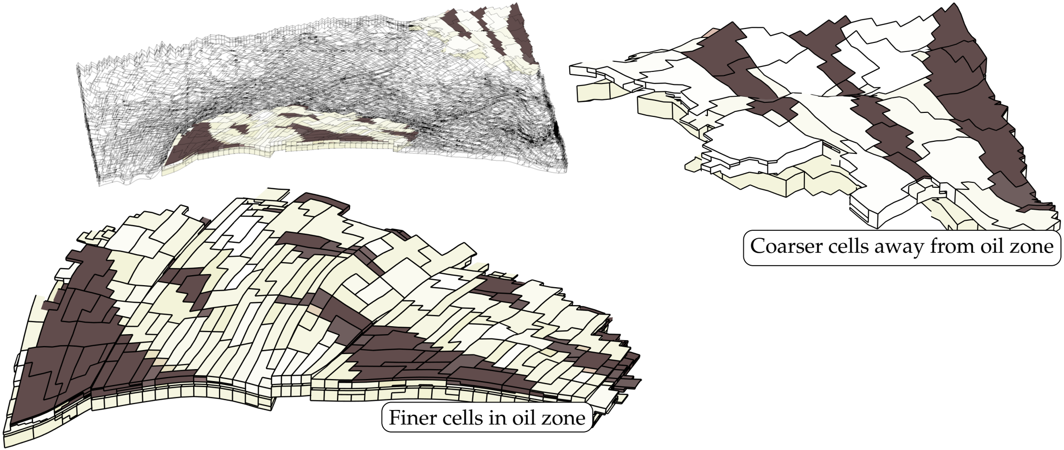

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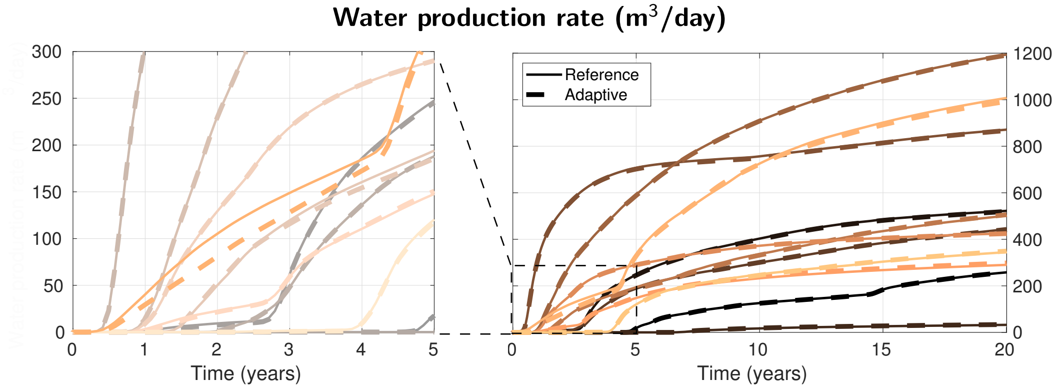
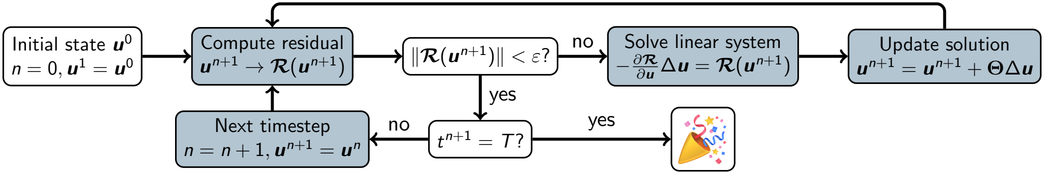


Very close match between reference and dynamic solution in all production wells

Concluding Remarks

Goal: Develop accurate and efficient discretizations and solution strategies for reservoir simulation by exploiting physical and mathematical structures of the underlying problem

- Has been achieved by working with a variety of topics
  - Unstructured gridding algorithms and consistent discretizations for complex reservoirs
  - Efficient iterative linear solver with elliptic multiscale preconditioning
  - Nonlinear transport solver with sophisticated adaptive trust-region damping
  - Localized nonlinear transport solvers based on reordering, with dynamic coarsening
- Results published in nine scientific papers (4 journal, 3 conference, 2 in preparation)



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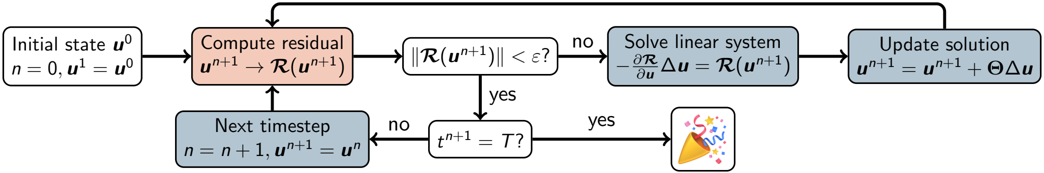

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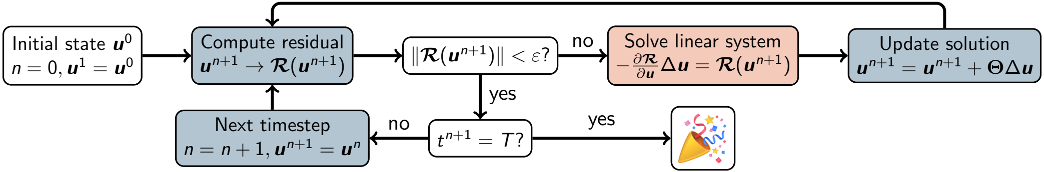

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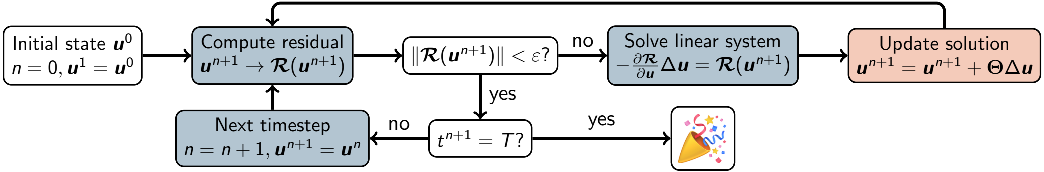

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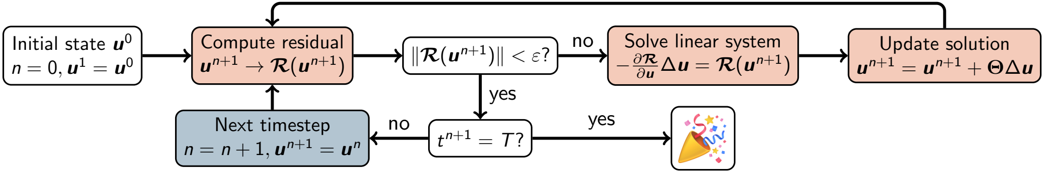

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Reproducible and transparent research

- Open-source software development (Matlab Reservoir Simulation Toolbox, `www.mrst.no`)
- Methods tested on challenging and realistic problems, with variety of parameters
  - Reported cases when methods do *not* work well

Acknowledgements

Thanks to my supervisors

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To my colleagues

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