Efficient Nonlinear Solution Strategies for Geothermal Energy Simulation

Øystein Klemetsdal and Odd Andersen, Applied Computational Sciences, SINTEF Digital

SIAM Geosciences 2023, June 19 – 22, Bergen, Norway

Presentation outline

- Motivation
- Governing equations and discretization
- Nonlinear domain decomposition
- Numerical examples
- Concluding remarks

SINTEF

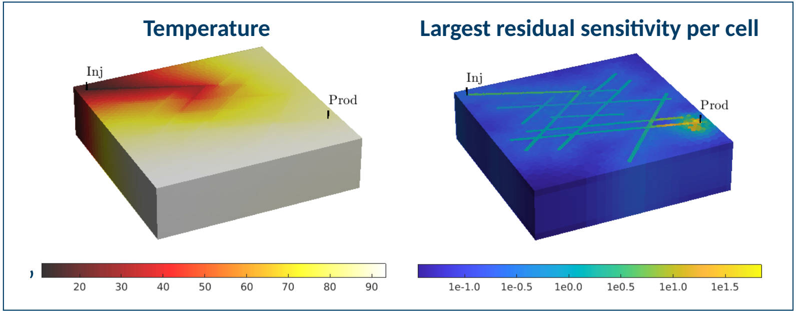
Motivation

- Geothermal energy systems often exhibit very complex geology
  - strong and abrupt variations in geological properties
  - intertwined faults/fracture networks and multiple long, deviating well trajectories
- Governing equations have very different timescales
  - heat rapidly advected through wellbores/fractures, slowly conducted through solid rock
- Mass/energy are strongly coupled through temperature/pressure-dependent density

$\rightarrow$ Strongly coupled nonlinear systems that are challenging to solve numerically

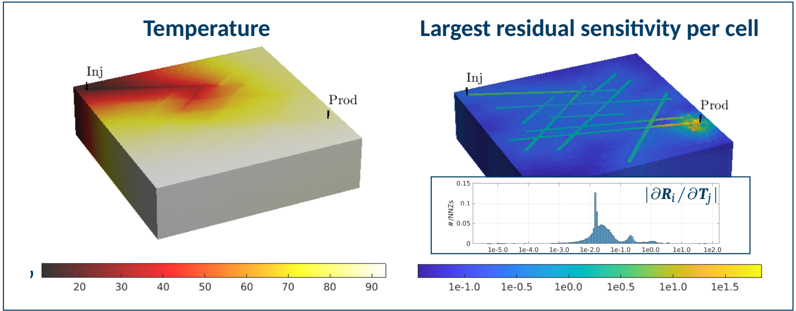
Motivation

But: strong nonlinearities are **chiefly localized in space**



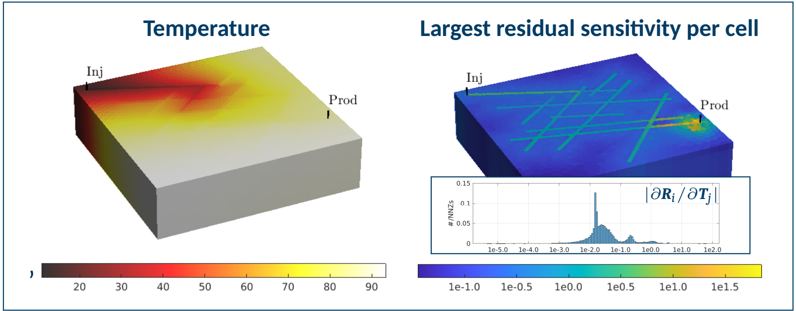
Motivation

But: strong nonlinearities are **chiefly localized in space**



Motivation

But: strong nonlinearities are **chiefly localized in space**



Here: exploit this locality to devise efficient domain-decomposition nonlinear solutions strategies applicable to practical simulation of geothermal energy systems

Geothermal simulation software and selected references

- There exists a myriad of excellent software capable of simulating geothermal systems
  - DARTS (TU Delft), AD-GPRS (Stanford), CSMP++ (ETH Zürich/Uni Melbourne ++), PorePy (UiB), TOUGH2 (LBNL), Dumu^x (Uni Stuttgart), JutulDarcy, **MRST** (SINTEF), etc.
- Many approaches to tackling various challenging aspects of geothermal simulation
  - Sequential splitting schemes (Weis et al. 2014; Wong, Kwok, et al. 2019)
  - Efficient, operator-based linearization (Wang et al. 2020)
  - Fracture modelling (HosseiniMehr et al. 2020)
  - Adaptive mesh refinement (Salinas et al. 2021)
  - Peaceman-type formulations for FEM in geothermal simulations (Yapparova et al. 2022)
  - Negative compressibility (Wong, Horne, and Tchelepi 2018)
  - Domain decomposition targeting local/unbalanced nonlinearities (Wong 2018)

Presentation outline

- Motivation
- Governing equations and discretization
- Nonlinear domain decomposition
- Numerical examples
- Concluding remarks

Governing equations and discretization

Conservation of mass – finite volumes in space, implicit backward Euler in time

$$\boldsymbol{R}_w^{n+1} = \frac{1}{\Delta t^n}\left(\boldsymbol{M}_w{}^{n+1} - \boldsymbol{M}_w^n\right) + \texttt{div}(\boldsymbol{V}_w{}^{n+1}) - \boldsymbol{Q}_w{}^{n+1} = 0$$
$$\boldsymbol{V}_w = -\texttt{upw}(\boldsymbol{\rho_w}/\boldsymbol{\mu_w})[\texttt{Kgrad}(\boldsymbol{p}) - g\texttt{favg}(\boldsymbol{\rho_w})\texttt{Kgrad}(\boldsymbol{z})]$$

Governing equations and discretization

Conservation of mass – finite volumes in space, implicit backward Euler in time

Mass · Flux · Sources/sinks

$$\boldsymbol{R}_w^{n+1} = \frac{1}{\Delta t^n}(\boldsymbol{M}_w{}^{n+1} - \boldsymbol{M}_w^n) + \mathtt{div}(\boldsymbol{V}_w{}^{n+1}) - \boldsymbol{Q}_w{}^{n+1} = 0$$

$$\boldsymbol{V}_w = -\mathtt{upw}(\boldsymbol{\rho_w}/\boldsymbol{\mu_w})[\mathtt{Kgrad}(\boldsymbol{p}) - g\mathtt{favg}(\boldsymbol{\rho_w})\mathtt{Kgrad}(\boldsymbol{z})]$$

Darcy's law

Governing equations and discretization

Conservation of mass – finite volumes in space, implicit backward Euler in time

Mass · Flux · Sources/sinks · Darcy's law

$$\boldsymbol{R}_w^{n+1} = \frac{1}{\Delta t^n}\left(\boldsymbol{M}_w{}^{n+1} - \boldsymbol{M}_w^n\right) + \mathtt{div}\left(\boldsymbol{V}_w{}^{n+1}\right) - \boldsymbol{Q}_w{}^{n+1} = 0$$

$$\boldsymbol{V}_w = -\mathtt{upw}(\boldsymbol{\rho}_w/\boldsymbol{\mu}_w)\left[\mathtt{Kgrad}(\boldsymbol{p}) - g\,\mathtt{favg}(\boldsymbol{\rho}_w)\mathtt{Kgrad}(\boldsymbol{z})\right]$$

- `div`, `upw`, `favg`: Discrete divergence, upwind, and face average operators
- `Kgrad`: discrete permeability-gradient operator $K\nabla$
  - linear/nonlinear two-point, multipoint, mimetic, etc.
  - Here: linear two-point flux approximation (comparison: Klemetsdal et. al. 2020, FVCA-IX)

Governing equations and discretization

Conservation of mass – finite volumes in space, implicit backward Euler in time

Mass · Flux · Sources/sinks · Darcy's law

$$\boldsymbol{R}_w^{n+1} = \frac{1}{\Delta t^n}\left(\boldsymbol{M}_w{}^{n+1} - \boldsymbol{M}_w^n\right) + \mathtt{div}(\boldsymbol{V}_w{}^{n+1}) - \boldsymbol{Q}_w{}^{n+1} = 0$$

$$\boldsymbol{V}_w = -\mathtt{upw}(\boldsymbol{\rho_w}/\boldsymbol{\mu_w})\left[\mathtt{Kgrad}(\boldsymbol{p}) - g\,\mathtt{favg}(\boldsymbol{\rho_w})\mathtt{Kgrad}(\boldsymbol{z})\right]$$

- `div`, `upw`, `favg`: Discrete divergence, upwind, and face average operators
- `Kgrad`: discrete permeability-gradient operator $K\nabla$
  - linear/nonlinear two-point, multipoint, mimetic, etc.
  - Here: linear two-point flux approximation (comparison: Klemetsdal et. al. 2020, FVCA-IX)

Governing equations and discretization

Conservation of energy – finite volumes in space, implicit backward Euler in time

$$
\begin{aligned}
\boldsymbol{R}_h^{n+1} = \tfrac{1}{\Delta t^n}\big([\boldsymbol{M}_w\boldsymbol{u}_w + \boldsymbol{M}_r\boldsymbol{u}_r]^{n+1} - [\boldsymbol{M}_w\boldsymbol{u}_w + \boldsymbol{M}_r\boldsymbol{u}_r]^{n}\big)& \\
+ \texttt{div}\big([\boldsymbol{V}_w\boldsymbol{h}_w]^{n+1} + \boldsymbol{H}^{n+1}\big) - [\boldsymbol{Q}_w\boldsymbol{h}_w]^{n+1} - \boldsymbol{Q}_h^{n+1} &= 0 \\
\boldsymbol{H} = -\Theta\texttt{grad}(\boldsymbol{T})&
\end{aligned}
$$

Governing equations and discretization

Conservation of energy – finite volumes in space, implicit backward Euler in time

Fluid energy

Rock energy

$$\boldsymbol{R}_h^{n+1} = \frac{1}{\Delta t^n}\left([\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n+1} - [\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n}\right) + \texttt{div}\left([\boldsymbol{V}_w \boldsymbol{h}_w]^{n+1} + \boldsymbol{H}^{n+1}\right) - [\boldsymbol{Q}_w \boldsymbol{h}_w]^{n+1} - \boldsymbol{Q}_h^{n+1} = 0$$

$$\boldsymbol{H} = -\Theta \texttt{grad}(\boldsymbol{T})$$

Fourier's law

Advection

Conduction

Governing equations and discretization

Conservation of energy – finite volumes in space, implicit backward Euler in time

Fluid energy Rock energy

$$\boldsymbol{R}_h^{n+1} = \frac{1}{\Delta t^n}\left([\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n+1} - [\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n}\right) + \texttt{div}\left([\boldsymbol{V}_w \boldsymbol{h}_w]^{n+1} + \boldsymbol{H}^{n+1}\right) - [\boldsymbol{Q}_w \boldsymbol{h}_w]^{n+1} - \boldsymbol{Q}_h^{n+1} = 0$$

$$\boldsymbol{H} = -\Theta\texttt{grad}(\boldsymbol{T})$$

Fourier's law Advection Conduction

- $\Theta\texttt{grad}$: discrete thermal conductivity/gradient operator $\Lambda\nabla$

Governing equations and discretization

Conservation of energy – finite volumes in space, implicit backward Euler in time

Fluid energy | Rock energy

$$\boldsymbol{R}_h^{n+1} = \frac{1}{\Delta t^n}\left([\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n+1} - [\boldsymbol{M}_w \boldsymbol{u}_w + \boldsymbol{M}_r \boldsymbol{u}_r]^{n}\right) + \mathtt{div}\left([\boldsymbol{V}_w \boldsymbol{h}_w]^{n+1} + \boldsymbol{H}^{n+1}\right) - [\boldsymbol{Q}_w \boldsymbol{h}_w]^{n+1} - \boldsymbol{Q}_h^{n+1} = 0$$

$$\boldsymbol{H} = -\Theta\mathtt{grad}(\boldsymbol{T})$$

Fourier's law | Advection | Conduction

- $\Theta\mathtt{grad}$: discrete thermal conductivity/gradient operator $\Lambda\nabla$
- Moreover: multisegment wellbore and discrete fracure modelling

  "Modelling and optimization of shallow geothermal energy storage" (Klemetsdal et. al, 2023 (in review))

Presentation outline

- Motivation
- Governing equations and discretization
- Nonlinear domain decomposition
- Numerical examples
- Concluding remarks

Nonlinear domain decomposition

- Partition domain into non-overlapping subdomains (here: two for simplicity)

$$\boldsymbol{R}(\boldsymbol{u}) = (\boldsymbol{R}_1(\boldsymbol{u}_1, \boldsymbol{u}_2), \boldsymbol{R}_2(\boldsymbol{u}_1, \boldsymbol{u}_2)) = 0$$

- Additive Schwarz: define solution operator $\mathcal{S}^a(\boldsymbol{u}) = (\mathcal{S}_1^a(\boldsymbol{u}), \mathcal{S}_2^a(\boldsymbol{u}))$, where

$$\boldsymbol{R}_1\big(\mathcal{S}_1^a(\boldsymbol{u}), \boldsymbol{u}_2\big) = 0, \quad \text{and} \quad \boldsymbol{R}_2\big(\boldsymbol{u}_1, \mathcal{S}_2^a(\boldsymbol{u})\big) = 0$$

- Multiplicative Schwarz: define solution operator $\mathcal{S}^m(\boldsymbol{u}) = (\mathcal{S}_1^m(\boldsymbol{u}), \mathcal{S}_2^m(\boldsymbol{u}))$, where

$$\boldsymbol{R}_1\big(\mathcal{S}_1^m(\boldsymbol{u}), \boldsymbol{u}_2\big) = 0, \quad \text{and} \quad \boldsymbol{R}_2\big(\mathcal{S}_1^m(\boldsymbol{u}), \mathcal{S}_2^m(\boldsymbol{u})\big) = 0$$

Nonlinear domain decomposition

- Partition domain into non-overlapping subdomains (here: two for simplicity)

$$\boldsymbol{R}(\boldsymbol{u}) = (\boldsymbol{R}_1(\boldsymbol{u}_1, \boldsymbol{u}_2), \boldsymbol{R}_2(\boldsymbol{u}_1, \boldsymbol{u}_2)) = 0$$

- Additive Schwarz: define solution operator $\mathcal{S}^a(\boldsymbol{u}) = (\mathcal{S}_1^a(\boldsymbol{u}), \mathcal{S}_2^a(\boldsymbol{u}))$, where

$$\boldsymbol{R}_1\left(\mathcal{S}_1^a(\boldsymbol{u}), \boldsymbol{u}_2\right) = 0, \quad \text{and} \quad \boldsymbol{R}_2\left(\boldsymbol{u}_1, \mathcal{S}_2^a(\boldsymbol{u})\right) = 0$$

- Multiplicative Schwarz: define solution operator $\mathcal{S}^m(\boldsymbol{u}) = (\mathcal{S}_1^m(\boldsymbol{u}), \mathcal{S}_2^m(\boldsymbol{u}))$, where

$$\boldsymbol{R}_1\left(\mathcal{S}_1^m(\boldsymbol{u}), \boldsymbol{u}_2\right) = 0, \quad \text{and} \quad \boldsymbol{R}_2\left(\mathcal{S}_1^m(\boldsymbol{u}), \mathcal{S}_2^m(\boldsymbol{u})\right) = 0$$

Equivalent, fixed-point formulation of $\boldsymbol{R}(\boldsymbol{u}) = 0$

Find $\boldsymbol{u}$ so that $\boldsymbol{u} = \mathcal{S}(\boldsymbol{u})$, or $\boldsymbol{F}(\boldsymbol{u}) \equiv \boldsymbol{u} - \mathcal{S}(\boldsymbol{u}) = 0$

Nonlinear domain decomposition preconditioning

Equivalent, fixed-point formulation of $\boldsymbol{R}(\boldsymbol{u}) = 0$

Find $\boldsymbol{u}$ so that $\boldsymbol{u} = \mathcal{S}(\boldsymbol{u})$, or $\boldsymbol{F}(\boldsymbol{u}) \equiv \boldsymbol{u} - \mathcal{S}(\boldsymbol{u}) = 0$

- Fixed-point schemes tend to have poor convergence properties
  - Acceleration: Aitken, Anderson, quasi-Newton (Jiang and Tchelepi 2019)

Nonlinear domain decomposition preconditioning

Equivalent, fixed-point formulation of $\boldsymbol{R}(\boldsymbol{u}) = 0$

Find $\boldsymbol{u}$ so that $\boldsymbol{u} = \mathcal{S}(\boldsymbol{u})$, or $\boldsymbol{F}(\boldsymbol{u}) \equiv \boldsymbol{u} - \mathcal{S}(\boldsymbol{u}) = 0$

- Fixed-point schemes tend to have poor convergence properties
  - Acceleration: Aitken, Anderson, quasi-Newton (Jiang and Tchelepi 2019)
- **As a nonlinear preconditionder:** apply Newton's method directly to $\boldsymbol{F}(\boldsymbol{u})$
  $\rightarrow$ Additive/Multiplicative Schwarz Preconditioned Exact Newton Method (ASPEN/MSPEN)
  (Cai and Keyes 2002; Liu, Keyes, and Sun 2013; Wong 2018, ...)

$$\boldsymbol{u}^{k+1} = \boldsymbol{u}^k + \Delta\boldsymbol{u}, \quad -\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}}\Delta\boldsymbol{u} = \boldsymbol{F}(\boldsymbol{u}^k), \quad \text{where} \quad \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}} = \boldsymbol{I} - \begin{bmatrix} \frac{\partial \mathcal{S}_1}{\partial \boldsymbol{u}} \\ \frac{\partial \mathcal{S}_2}{\partial \boldsymbol{u}} \end{bmatrix}$$

  - Challenge: $\boldsymbol{F}$ implicitly defined through operator $\mathcal{S}$ – how to compute $\partial\boldsymbol{F}/\partial\boldsymbol{u}$?

Nonlinear domain decomposition preconditioning

- Use that $\boldsymbol{R}_1(\mathcal{S}_1^{a/m}(\boldsymbol{u}), \boldsymbol{u}_2) = 0$ to find

$$\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}} = \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1}\frac{\partial \mathcal{S}_1^{a/m}}{\partial \boldsymbol{u}} + \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_2}\frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{u}} = 0 \Rightarrow \frac{\partial \mathcal{S}_1^{a/m}}{\partial \boldsymbol{u}} = -\left(\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1}\right)^{-1}\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_2}\frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{u}}$$

Nonlinear domain decomposition preconditioning

- Use that $\boldsymbol{R}_1(\mathcal{S}_1^{a/m}(\boldsymbol{u}), \boldsymbol{u}_2) = 0$ to find

$$\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}} = \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1}\frac{\partial \mathcal{S}_1^{a/m}}{\partial \boldsymbol{u}} + \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_2}\frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{u}} = 0 \Rightarrow \frac{\partial \mathcal{S}_1^{a/m}}{\partial \boldsymbol{u}} = -\left(\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1}\right)^{-1}\frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_2}\frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{u}}$$

- Additive: analogous derivation for $\partial \mathcal{S}_2^a / \partial \boldsymbol{u}$
- Multiplicative: Use that $\boldsymbol{R}_2(\mathcal{S}_1^m(\boldsymbol{u}), \mathcal{S}_1^m(\boldsymbol{u}_2)) = 0$ to find

$$\frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}} = \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_1}\frac{\partial \mathcal{S}_1^{m}}{\partial \boldsymbol{u}} + \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2}\frac{\partial \mathcal{S}_2^{m}}{\partial \boldsymbol{u}} = 0 \Rightarrow \frac{\partial \mathcal{S}_2^{m}}{\partial \boldsymbol{u}} = -\left(\frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2}\right)^{-1}\frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_1}\frac{\partial \mathcal{S}_1^{m}}{\partial \boldsymbol{u}}$$

- Natural extension to N subdomains

Nonlinear domain decomposition preconditioning

- Jacobian $\partial \boldsymbol{F}/\partial \boldsymbol{u}$ generally dense $\rightarrow$ expensive to build, challenging to precondition
- Breakdown of Jacobian blocks reveals that[1]

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}} = \boldsymbol{D}^{-1} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}$$

- Where $\boldsymbol{D}$ is a block matrix

$$\text{Additive:} \quad \boldsymbol{D} = \begin{bmatrix} \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1} & 0 \\ 0 & \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2} \end{bmatrix}, \quad \text{Multiplicative:} \quad \boldsymbol{D} = \begin{bmatrix} \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1} & 0 \\ \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_1} & \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2} \end{bmatrix}$$

[1] "A numerical study of ASPEN (...)", Øystein Klemetsdal et al. 2021

Nonlinear domain decomposition preconditioning

- Jacobian $\partial \boldsymbol{F}/\partial \boldsymbol{u}$ generally dense $\rightarrow$ expensive to build, challenging to precondition
- Breakdown of Jacobian blocks reveals that[1]

$$\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}} = \boldsymbol{D}^{-1}\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}$$

- Where $\boldsymbol{D}$ is a block matrix

$$\text{Additive:}\quad \boldsymbol{D} = \begin{bmatrix} \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1} & 0 \\ 0 & \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2} \end{bmatrix}, \quad \text{Multiplicative:}\quad \boldsymbol{D} = \begin{bmatrix} \frac{\partial \boldsymbol{R}_1}{\partial \boldsymbol{u}_1} & 0 \\ \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_1} & \frac{\partial \boldsymbol{R}_2}{\partial \boldsymbol{u}_2} \end{bmatrix}$$

$\rightarrow$ Can interpret linearized system as

Original problem Jacobian (almost)

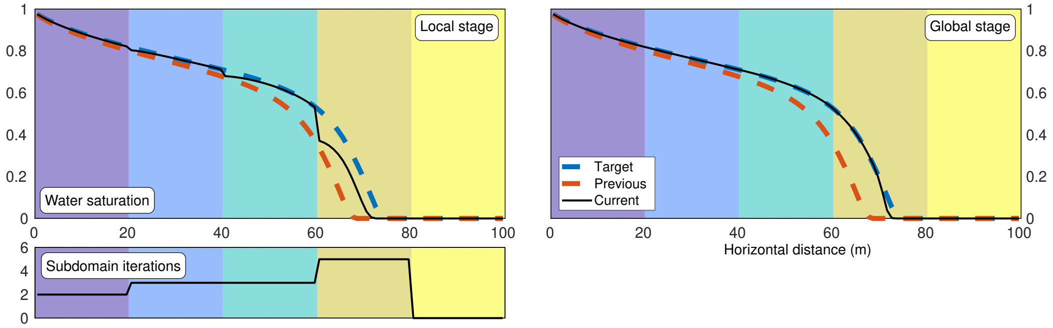
$$-\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}}\Delta \boldsymbol{u} = \boldsymbol{F}(\boldsymbol{u}) \quad \Longleftrightarrow \quad -\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\Delta \boldsymbol{u} = \boldsymbol{D}\boldsymbol{F}(\boldsymbol{u}).$$

  - We're back on home ground – we know what preconditioners to use!

[1] "A numerical study of ASPEN (...)", Øystein Klemetsdal et al. 2021

Nonlinear domain decomposition preconditioning

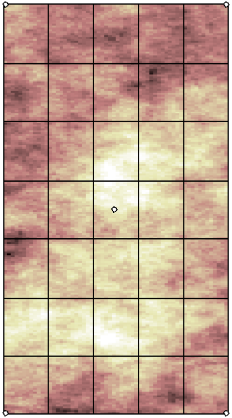
Illustrating example: 1D Buckley-Leverett displacement with five subdomains



Presentation outline

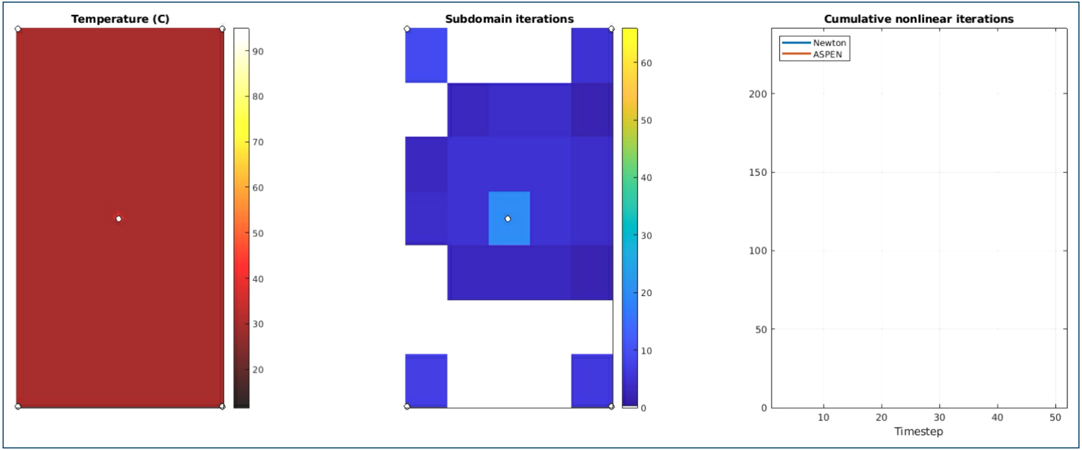
- Motivation
- Governing equations and discretization
- Nonlinear domain decomposition
- **Numerical examples**
- Concluding remarks

Example: Thermal storage in subset of SPE10 Model 2

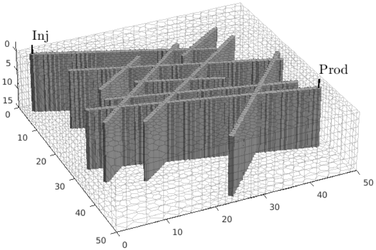


- Layer 10 of SPE10 Model 2 (Christie and Blunt 2001)
  - Charge: four months at 5 l/s and 90 °C through center well
  - Discharge: four months at 5 l/s and 10 °C through corner wells
- Compare two nonlinear solution strategies
  - Standard Newton
  - ASPEN with 5×7 subdomains

Example: Thermal storage in subset of SPE10 Model 2



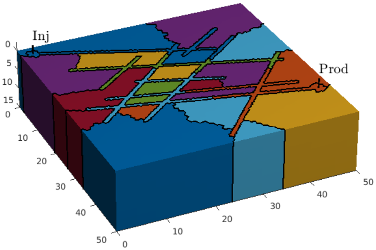
Example: Enhanced Geothermal System (EGS)[1]



- Fractured, low-perm, high-temp, subsurface rock
- Water circulates through the fracture network
  $\rightarrow$ Fractures act as fins of a heat exchanger
- Here: artificial network in confined, insulated box
- Injection temp: 10 °C, reservoir temp: 95 °C

[1] From "Simulation of Geothermal Systems Using MRST", Collignon, Øystein Klemetsdal, and Møyner 2021

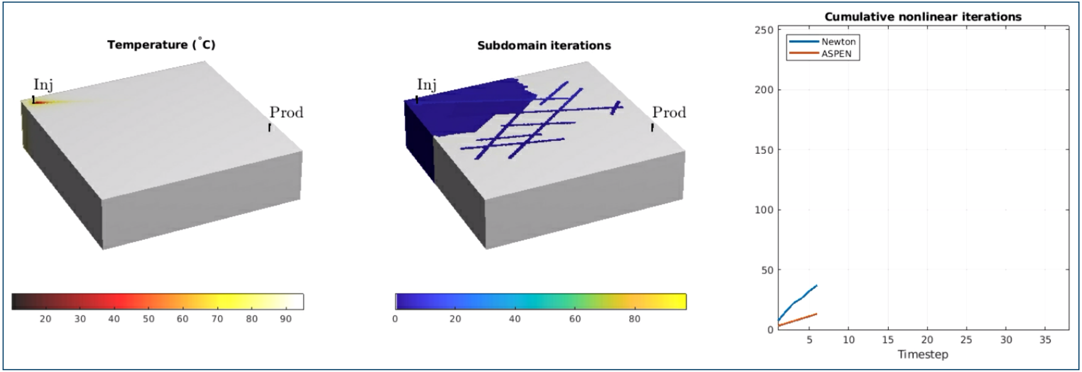
Example: Enhanced Geothermal System (EGS)[1]



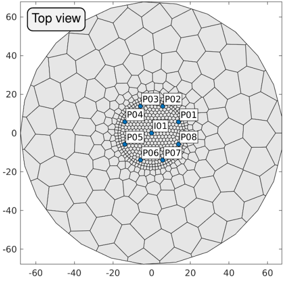
- Fractured, low-perm, high-temp, subsurface rock
- Water circulates through the fracture network
  $\rightarrow$ Fractures act as fins of a heat exchanger
- Here: artificial network in confined, insulated box
- Injection temp: 10 °C, reservoir temp: 95 °C
- Compare Newton and ASPEN
  - Three subdomains with fractures + wells only
  - 21 subdomains the for the remaining matrix

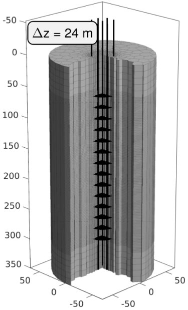
[1] From "Simulation of Geothermal Systems Using MRST", Collignon, Øystein Klemetsdal, and Møyner 2021

Example: Enhanced Geothermal System (EGS)



Example: Cyclic charging of fractured reservoir [1]

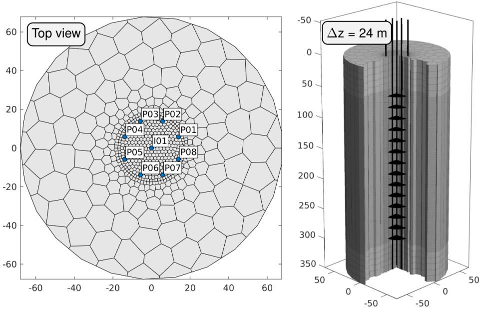




- Fractured, 300 m deep reservoir
- Injector circled by eight producers
- Five cycles of charging and discharging through center well
  - Charge: 6 months, 50 l/s, 140 °C
  - Discharge: 6 months, 50 l/s, 10 ° C
- ”Spaghetti topology” – significant buoyancy effects inside wellbore
  - Need mulitsegment well formulation

[1] Adapted from “Modeling and Optimization of Shallow Geothermal Heat Storage”, Ø. Klemetsdal et al. 2022

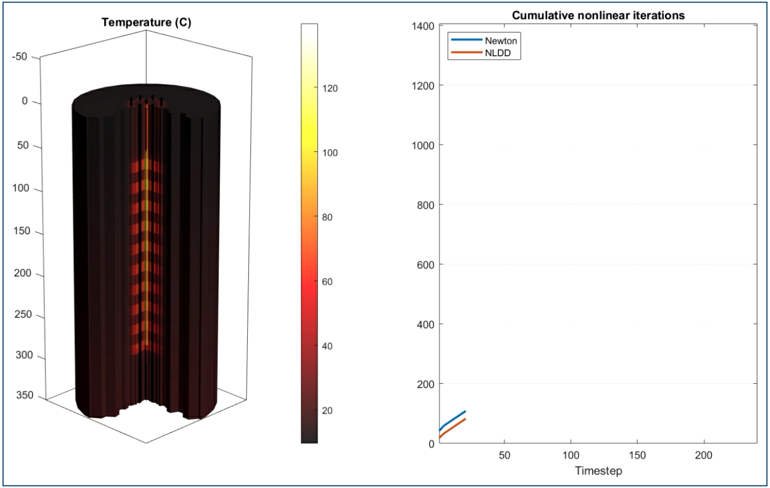
Example: Cyclic charging of fractured reservoir [1]



- Compare Newton with multiplicative field-split-type strategy (NLDD):
    1. Solve conservation equations in wellbore with fixed reservoir properties
    2. Solve conservation equations in reservoir with fixed wellbore properties
    3. Do a single Newton step
    4. If not converged, go to 1

[1] Adapted from "Modeling and Optimization of Shallow Geothermal Heat Storage", Ø. Klemetsdal et al. 2022

Example: Cyclic charging of fractured reservoir



Concluding remarks

Conclusions

- Efficient Nonlinear solution strategies applicable to realistic geothermal problems
- Very robust with respect to timestep length
- Significant reduction in nonlinear iterations for examples considered
  - SPE10 layer 10: 36 %, EGS: 65 %, Cyclic storage: 36 %
- But: local solves introduces additional cost that **may prohibit speedup**
  - Local stage is embarrassingly parallel, efficient implementation possible
  - Recent work indicates that adaptive strategies can be very beneficial

    An Adaptive Newton–ASPEN Solver for Complex Reservoir Models, Lie, Møyner, and Ø. A. Klemetsdal 2023

SINTEF

Concluding remarks

Conclusions

- Efficient Nonlinear solution strategies applicable to realistic geothermal problems
- Very robust with respect to timestep length
- Significant reduction in nonlinear iterations for examples considered
  - SPE10 layer 10: 36 %, EGS: 65 %, Cyclic storage: 36 %
- But: local solves introduces additional cost that **may prohibit speedup**
  - Local stage is embarrassingly parallel, efficient implementation possible
  - Recent work indicates that adaptive strategies can be very beneficial

    An Adaptive Newton–ASPEN Solver for Complex Reservoir Models, Lie, Møyner, and Ø. A. Klemetsdal 2023

Future work

- Combined multiplicative/adaptive preconditioning for wells/fractures/matrix
- Combine with dynamic, locally adaptive timestepping

MRST
TRANSFORMING RESEARCH

SINTEF

Technology for a
better society